

# Optimization Method of the Total Weighed Holding Time in a P2P Network

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**Abstract**— In this paper the formulation of optimization problem of the total weighed holding time is given by working out of mathematical model in peer-to-peer networks communication. The mathematical model Optimization constructed on the schedules theory. The described models and the schedules theory methods allow to display adequately structure and a P2P network function and to consider their dynamics, restriction of previous information, shifts, interruptions and other typical technological decisions that can be used for increase of a network productivity and granting quality of services QoS, QoE.

**Keywords:** peer-to-peer, mathematical model, network, the schedules theory, fragments delays, Weighed Holding Time, QoS.

## Introduction

Applications Peer-To-Peer (P2P) have become very popular among Internet users. P2P technologies improve scalability of the system at low costs of implementation. Unlike traditional client-server architectures, peers in the network act as a client (leach-downloading) and a server (sid-distributing). Peers do not only download a file from the network, but also distribute the loaded file to other users in the network. Parts of files are exchanged via direct connections between the peers.

Another important property of peer-to-peer networks is the ability to order fragments, interruptions, entries and exits from the network at any time.

Practical use of P2P network is constantly expanding, and user traffic according to Cisco Systems in 2013 reached 60 peta-bytes / month [1]. The ability to organize distributed computing gives important practical use enabling these networks to compete with the performance of modern supercomputers. A large portion of the tasks accounts on the organization of online games (MMOG). However, according to the same data [1], about half of the traffic of peer-to-peer networks accounts on the organization of files sharing P2P-TV, which attracts special attention to this technology.

Architecture of P2P networks may be structured or non-structured [2]. In structured systems P2P-HQTV overlay structure typically represents semi-determined topology, which

causes difficulties in restructuring at the connection or disconnection of another peer. A distinctive feature of unstructured architecture of P2P-TV systems is that the overlay topology is completely separated from the topology of distribution. The result is a mesh-topology allowing for the implementation of a high degree of dynamism and decentralization.

P2P networks as private, overlay file-sharing networks have a number of specific features in relation to the total IP-based network. Thus, in the famous file-sharing applications (Bit Torrent), optimization of the network performance is reduced to finding the "best" peer, from which the content is downloaded. Let us point to other important tasks to improve the network performance that need to be addressed in parallel in real time [3,4,5]:

- algorithms to plan elements;
- peer selection with low delays,
- ensuring interaction between the dynamics of the overlay and IP-routing.

Well-known systemic quality criteria: QoS and QoE directly depend on quality of service tasks fulfillment, however, do not always allow to obtain the solution in full. That is why, many papers addressing research of peer-to-peer networks are very specific [4,5,6,7]. Our objective is to attempt to obtain a systemic solution, optimal for a delay minimum criterion upon service completion subject to a number of standard restrictions.

## Task:

Let us assume that P2P-TV network provides simultaneous viewing of  $m$  -programs

$m \in M$  . One of these program is viewed by  $l$  - subscribers (peers)  $l \in L$  . At any moment each

of  $L$ -peers can stop servicing, and then also start it. Each peer has  $N = \{1, 2, \dots, n\}$  requirements represented as separate  $J$ -fragments of the downloaded content.

Duration of servicing by the network of each of the listed  $J$ -fragments is  $p_j$ , where  $j \in N$ . Process of content downloading represents duration of procedures ensuring partial order, which is ultimately determined by the precedence restrictions. If restrictions have a directive term to complete service for  $J$ -fragment  $d_j$ , then the difference between the moment of completion  $C_j$  and the directive moment  $d_j$  at  $C_j > d_j$  shall be deemed a delay in the service or service with advance at  $C_j < d_j$ .

Let us assume that each user has a buffer that can contain the whole set of requirements  $N = \{1, 2, \dots, n\}$ , where there is  $n$ -number of storage places of  $n$ -fragment obtained in the

**Task Formulation for Total Weighted Holding Time Minimization.**

Let us assume, that when  $N$  requirement is fulfilled, not more than one fragment is received on a terminal at a time. For every  $j$  fragment belonging to a range of  $N = \{1, 2, \dots, n\}$ , the following parameters are preset:

- $p_j$ - is a  $j$ -fragment holding duration,  $0 < p_j^L \leq p_j^U$ ; where  $L$  and  $U$  indexes refer to lower and upper service limits, respectively. As a rule,  $p_j$  holding duration turns out to be unknown until completion of such  $j$ -holding. Random nature of  $p_j$  is determined by difference in parameters of a download line and technologies applied.

- every  $j$ -requirement is associated with a weight coefficient  $w_j > 0$ . Values of weight coefficients  $w_j$  cannot be the same, first of all, because importance of further fragments by the end of the download increases, that is why  $w_n \geq w_{n-1} \geq \dots w_1$ .

In order to find solutions to the objective, let's use outcomes of the Scheduling Theory [8,10,11], which is confined to construction of a

previous steps, plus one place intended for obtaining another new portion of data. In accordance with the boot rule, after the server randomly chose a peer for the initial boot, it finds the missing portion of data in the network of another peer and tries to download it. Thus, as a result of differences in channel capacity and duration of loading and receiving of a buffer, there may be cases of violation of the partial order [8].

To account for the precedence restrictions in the service, let us define the binary relation of the strict order  $\rightarrow$  on the set of requirements  $N$ . Requirement  $i$  is a predecessor of requirement  $j$ , and requirement  $j$  - follower in relation to  $i$ . Relation  $\rightarrow$  is set by the oriented circuit-free graph  $G$ , the set of peaks (nodes) of which is identified with a set of requirements, and the route from peak  $i$  to peak  $j$  exists, when  $i \rightarrow j$ .

function determined by multiple exchanges, and to development of optimization algorithms.

At every  $n$  - stage of holding, various implementation times  $p_j$  of every  $j$ -holding are possible,  $j \in N$ . In practice,  $p_j$  value forms a denumerable set of variants from  $[p_j^L; p_j^U]$  range. Thus,  $k$ -dimensional vector may be considered

$$P_i^T = (p_1, p_2, \dots, p_k), p_j \in [p_j^L; p_j^U]. \quad (1)$$

$p_j$  values in general become known after every fragment is received:  $p_j = p_{ps}^{(j)}$ .

Let us assume, that  $P$  denotes a set of all vectors (1) of possible requirements holding duration

$$P_i^T = (p_1^{(l)}, p_2^{(k)}, \dots, p_n^{(k)}), k = 1, 2, \dots, k. \quad (2)$$

Vector (2) is a scenario of events,  $n$ -fragments sequence upon a certain peer's request.

$P$  is obviously a closed rectangle of negative numbers.

For Scheduling  $S$  a sequence of the content fragments being downloaded and implementation of scenario (2), an additive weighed linear function may be chosen as a criterion function:

1. Total weighted time spent on vector (2) receipt

$$f_i(C_i) = \sum_{i=1}^n w_i C_i, \quad (3)$$

where  $w_i$  is a weight coefficient determining importance of  $i$  fragment, including its holding time.

For different  $w_i$  values (3) of functions  $f_i(C_i)$  are represented as a straight line in the diagrams of Fig. 1a.

2. Total weighted number of fragments taken with a delay

$$f_i(C_i) = \sum_{i=1}^n w_i U_i,$$

Where  $U_i$  equals to zero if  $C_i \leq d_i$  - directive term (in the absence of delays), and equals to 1 at  $d_i \leq C_i$ .

Possible values of functions  $f_i(C_i)$  are shown in Fig. 1b.

3. Total weighted delay of fragments

$$f_i(C_i) = \sum_{i=1}^n w_i T_i, \quad (4)$$

where  $T_i = \max\{0, C_i - d_i\}$ . Values of function (5) are shown in Fig. 1c.

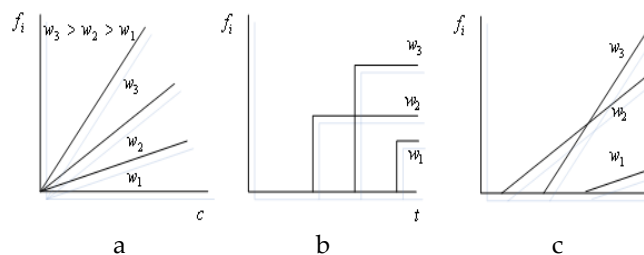


Fig. 1. Diagram of qualitative behavior of the criterion functions a-(3); b-(4); c-(5).

It should be borne in mind, that scenario (2) in general remains undetermined until completion of holding for all  $i$  and  $j$ . And, due to NP completeness, minimization of criterion functions (3), (4), (5) is impossible, at least, at the stage of computation and making of its proper schedule.

However, for the main technologies P2P-TV, when downloading specific files, differential

fragmentation is not typically used. This allows selection for all  $P_i$

$$P_i = P, \forall i. \quad (5)$$

With condition (5) vector parameters (1) can be considered known a priori. In this case we can get away from NP-completeness and solve the optimization task for time  $O(n \log n)$  [12]:

$$f_o(p) = \min_s \sum_{i=1}^n w_i G_i(p_i = p). \quad (6)$$

The Scheduling Theory includes many schedules of similar tasks subject to additional restrictions. So, there are known solutions based on dynamic programming method [12,13]. Among other methods of practical importance, there should be mentioned dichotomy method [13], robust techniques. A method based on sensitivity analysis [10] is considered to be prospective; it suggests two-stage adoption of decisions: stage of a priori off-line planning of  $k$  options, and a stage of online scheduling.

Due to different content downloading conditions, other random factors in a sequence of fragments received by a peer, precedence conditions may be disturbed, requiring ordering and corresponding exchanges in the fragment sequence. It imposes further restrictions on solution of the optimization problem. So, the previously chosen criterion (3) subject to the required exchange procedure  $\pi = (\pi_1, \pi_2, \dots, \pi_n!)$  shall take the following form:

$$J = \sum_{j=1}^n w_j C_j(\pi, P) = \min_{\pi_k} \left\{ \sum_{\pi} \sum_j w_j C_j(\pi_k, p) \right\} \quad (7)$$

The next important restriction frequent in P2P network operation is existence of interruptions caused by technological reasons during holding under high load or through effect of holdings of higher priorities [8].

Let us consider an example of weighted average holding start time. Let us assume that  $M$  duration of the downloaded file holding is known. Master data:

$I = \{i\}$  - is a set of fragments;

$P = \{p_i\}$  - is a set of holding times;

$W = \{w_i\}$  - is a set of requirement penalties.

Objective function:

$$F(X) = W^T * X \rightarrow \min .$$

For every pair of fragments  $i, j \in I$ , one of  $x_i - x_j \geq p_j$  conditions is applied, which corresponds to precedence  $j$  comparing to fragment  $i$  or  $x_j - x_i \geq p_i$ , when  $i$  precedes  $j$  fulfillment. Let us assume, that precedence coefficient is equal to  $y_{i,j} = 1$ , if  $i$  precedes  $j$ , and 0 in an opposite case. It allows recording the restriction system as follows:

$$(M + p_j)y_{ij} + (x_i - x_j) \geq p_j \quad (8)$$

$$(M + p_i)(1 - y_{ij}) + (x_j - x_i) \geq p_i . \quad (9)$$

Thus, the following objective function is obtained:

**Conclusion**

1. P2P-TV networks are intensively developing and become more and more popular both

in the entertainment industry and in solving a number of professional tasks of the info communicational direction.

2. Existing technologies of P2P-TV networks allow finding solutions concerning optimization of available resources aimed at enhancement of the network performance and service qualities QoS, QoE.

3. Models and methods of the Schedule Theory are adequate for fulfillment of tasks on peering network quality criteria improvement. They enable to adequately depict both the structure, and functions of these networks, pay due regard to their dynamics, restrictions on precedence, exchange, interruption and other typical technological conditions.

$$F(X) = 4x_1 + 2x_2 + 2x_3 + 3x_4 \rightarrow \min$$

Optimal solution was obtained by means of Matlab package for four fragments, under the assumption, that

$$p = \{6, 2, 2, 3\} \quad W = \{4, 2, 2, 3\}$$

This solution is provided on figure (2) with a Gantt diagram in a form of sequence.

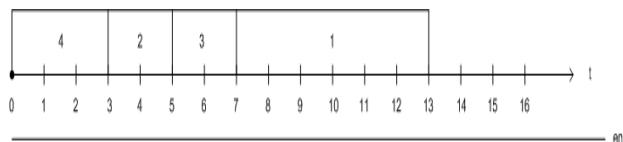


Fig.2. Optimal sequence of downloaded fragments

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